Ex1 :

1) $f(x) = ax + b$.

$$a = \frac{y_B - y_A}{x_B - x_A} = \frac{4-2}{1-(-2)} = \frac{2}{3}$$

$B \in C_f \Leftrightarrow f(x_B) = y_B \Leftrightarrow \frac{2}{3} \times x_B + b = y_B$

$\Leftrightarrow \frac{2}{3} \times 1 + b = 4 \Leftrightarrow \frac{2}{3} + b = 4 \Leftrightarrow b = 4 - \frac{2}{3}$.

donc $b = \frac{10}{3}$.

$\Rightarrow \boxed{f(x) = \frac{2}{3}x + \frac{10}{3}}$ ou $\boxed{f(x) = \frac{2x+10}{3}}$

2) $F(x) = \frac{2}{3}\frac{x^2}{2} + \frac{10}{3}x + k$ (k est une Constante) $k \in \mathbb{R}$

$\Rightarrow \boxed{F(x) = \frac{x^2}{3} + \frac{10}{3}x + k}$

3) a) $$\int_{-2}^{1} f(x)\,dx = \left[F(x)\right]_{-2}^{1} = F(1) - F(-2).$$

$$= \left[\frac{1^2}{3} + \frac{10}{3} \times 1\right] - \left[\frac{(-2)^2}{3} + \frac{10}{3}(-2)\right].$$

$$= \left(\frac{11}{3}\right) - \left(\frac{4-20}{3}\right) = \frac{11+16}{3} = \frac{27}{3} = 9.$$

b) En utilisant le quadrillage, on obtient :

$$\mathcal{A} = 2 \times 3 + \frac{3 \times 2}{2} = 6 + 3 = 9$$

Ex 2 :

1) $\int_{-4}^{-2} f(x)\,dx = \left(3+\frac{1}{2}\right) = 3{,}5$ u.a.

$\int_{-2}^{0} f(x)\,dx = 4 - \frac{\pi \times 1^2}{2} = \left(4-\frac{\pi}{2}\right)$ u.a.

$\int_{0}^{2} f(x)\,dx = \frac{2\times 2}{2} = 2$ u.a

2) $\int_{-4}^{2} f(x)\,dx = \left(3{,}5+4-\frac{\pi}{2}+2\right) = \left(9{,}5-\frac{\pi}{2}\right)$ u.a.

Ex 3 :

1) $\int_{1}^{2} f(x)\,dx = \int_{1}^{2} \frac{1}{x}\,dx = \left[\ln|x|\right]_1^2 = \ln 2 - \ln 1 = \ln 2$

2) $\int_{2}^{3} (1-f(x))\,dx$ ou bien $1-\int_{2}^{3} f(x)\,dx = 1-\left[\ln|x|\right]_2^3$

$= 1-(\ln 3-\ln 2)$

$= \boxed{1+\ln 2-\ln 3}$

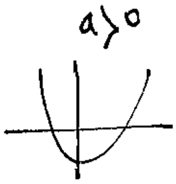
3) $\int_{0}^{1} (h(x)-g(x))\,dx = \int_{0}^{1} (x-x^2)\,dx = \left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = \left(\frac{1^2}{2}-\frac{1^3}{3}\right)-\left(\frac{0^2}{2}-\frac{0^3}{3}\right)$

$= \frac{1}{2}-\frac{1}{3} = \frac{1}{6}$

Ex 4 : $f(x) = x^2-2x-3$ $\quad \Delta = (-2)^2-4\times 1\times(-3) = 16$

$x_1 = \frac{-(-2)+\sqrt{16}}{2\times 1} = \frac{2+4}{2} = 3$ $\quad x_2 = \frac{-(-2)-\sqrt{16}}{2\times 1} = \frac{2-4}{2} = -1$

$a>0$



x	$-\infty$	-1	3	$+\infty$
$f(x)$	$+$	0 $-$	0	$+$
posit°	C est au dessus de (Ox)	C est en dessous		C est au dessus de (Ox)

croisement — C est en dessous — croisement

$A = \int_{-2}^{-1} f(x) + \int_{-1}^{3} -f(x)\,dx$

$= \left[\frac{x^3}{3}-x^2-3x\right]_{-2}^{-1} - \left[\frac{x^3}{3}-x^2-3x\right]_{-1}^{3}$

$= \left[\left(\frac{-1}{3}-1+3\right)-\left(\frac{-8}{3}-4+6\right)\right] - \left[\left(\frac{27}{3}-9-9\right)-\left(-\frac{1}{3}-1+3\right)\right].$

Ex5 :

a) $\int_1^2 (2x^5 - x^2 - 1)\,dx = \left[2\frac{x^6}{6} - \frac{x^3}{3} - x\right]_1^2 = \left[\frac{x^6}{3} - \frac{x^3}{3} - x\right]_1^2$

$$= \left(\frac{2^6}{3} - \frac{2^3}{3} - 2\right) - \left(\frac{1^6}{3} - \frac{1^3}{3} - 1\right).$$

b) $\int_0^{-1} (1-t^2)(2+3t)\,dt. = \int_0^{-1} (-3t^3 - 2t^2 + 3t + 2)\,dt.$

$$= \left[-3\frac{t^4}{4} - 2\frac{t^3}{3} + 3\frac{t^2}{2} + 2t\right]_0^{-1}$$

$$= \left(-\frac{3}{4} + \frac{2}{3} + \frac{3}{2} - 2\right) - (0).$$

c) $\int_2^5 \frac{2}{3}\,dx = \left[\frac{2}{3}x\right]_2^5 = \frac{2}{3}(5-2) = \frac{2}{3} \times 3 = 2.$

d) $\int_{-1}^3 \frac{1}{n}\,dx = \left[\frac{1}{n}x\right]_{-1}^3 = \frac{1}{n}(3-(-1)) = \frac{1}{n} \times 4 = \frac{4}{n}$

n est donc constante (ne dépend pas de x).

Ex6

a) $\int_0^1 \frac{1}{1+2x}\,dx = \frac{1}{2}\int_0^1 \frac{2}{1+2x}\,dx = \frac{1}{2}\left[\ln|1+2x|\right]_0^1$

$$= \frac{1}{2}\left(\ln(1+2) - \ln(1+0)\right)$$

$$= \boxed{\frac{1}{2}\ln 3}$$

b) $\int_1^e \frac{6x^2+4x-1}{x}\,dx = \int_1^e \left(\frac{6x^2}{x} + \frac{4x}{x} - \frac{1}{x}\right)dx$

$$= \int_1^e \left(6x + 4 - \frac{1}{x}\right)dx = \left[6\frac{x^2}{2} + 4x - \ln|x|\right]_1^e$$

$$= (3e^2 + 4e - \ln e) - (3\times 1^2 + 4\times 1 - \ln 1) = 3e^2 + 4e - 1 - 3 - 4$$

$$= \boxed{3e^2 + 4e - 8}$$

c) $\int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{1}{3}\int_0^1 \frac{3x^2}{1+x^3}\,dx$

$= \frac{1}{3}\left[\ln|1+x^3|\right]_0^1$

$= \frac{1}{3}\left(\ln(1+1^3) - \ln(1+0^3)\right) = \boxed{\frac{1}{3}\ln 2}$

d) $\int_1^4 \frac{1}{3t} - \frac{3}{t^2}\,dt = \int_1^4 \frac{1}{3t}\,dt - \int_1^4 \frac{3}{t^2}\,dt$

$= \frac{1}{3}\int_1^4 \frac{1}{t}\,dt - 3\int_1^4 \frac{1}{t^2}\,dt$

$= \frac{1}{3}\left[\ln|t|\right]_1^4 - 3\left[\frac{-1}{3t^3}\right]_1^4$

$= \frac{1}{3}(\ln 4 - \ln 1) - 3\left(\frac{-1}{3\times 4^3} - \frac{-1}{3\times 1^3}\right)$

$= \boxed{\frac{1}{3}\ln 4 + \frac{1}{4^3} - 1}$

fct primitive

$\frac{1}{x^n} \longrightarrow \frac{-1}{(n+1)\,x^{n+1}}$

$\frac{U'}{U} \longrightarrow \ln|U|$

$\frac{U'}{U^n} \longrightarrow \frac{-1}{(n+1)\,U^{n+1}}$